**To derive the exit velocity (Vo*Vo*​), exit temperature (To*To*​), and exit density (ρo*ρo*​) under the conditions Pc=Po*Pc*​=*Po*​ (constant pressure) and Ao=Ac=π(7.5)2*Ao*​=*Ac*​=*π*(7.5)2, we use the ideal gas law, conservation of mass, and energy equation. Here's the step-by-step derivation:**

**1. Given Conditions**

* **Inlet: Pc,Tc,Vc,ρc,Ac=π(7.5)2*Pc*​,*Tc*​,*Vc*​,*ρc*​,*Ac*​=*π*(7.5)2**
* **Exit: Po=Pc,Ao=Ac*Po*​=*Pc*​,*Ao*​=*Ac*​**
* **Heat added: q=ηc⋅LHV⋅FAR*q*=*ηc*​⋅LHV⋅FAR**

**2. Conservation of Mass (Continuity Equation)**

**ρcVcAc=ρoVoAo*ρc*​*Vc*​*Ac*​=*ρo*​*Vo*​*Ao*​**

**Since Ac=Ao*Ac*​=*Ao*​, this simplifies to:**

**ρcVc=ρoVo⇒Vo=ρcρoVc(1)*ρc*​*Vc*​=*ρo*​*Vo*​⇒*Vo*​=*ρo*​*ρc*​​*Vc*​(1)**

**3. Ideal Gas Law**

**At both inlet and exit (Pc=Po*Pc*​=*Po*​):**

**Pc=ρcRTcandPo=ρoRTo*Pc*​=*ρc*​*RTc*​and*Po*​=*ρo*​*RTo*​**

**Since Pc=Po*Pc*​=*Po*​, equate the two:**

**ρcTc=ρoTo⇒ρo=ρcTcTo(2)*ρc*​*Tc*​=*ρo*​*To*​⇒*ρo*​=*ρc*​*To*​*Tc*​​(2)**

**4. Substitute ρo*ρo*​ into Continuity Equation**

**From Equation (2):**

**Vo=ρcρcTcToVc=ToTcVc⇒Vo=VcToTc(3)*Vo*​=*ρc*​*To*​*Tc*​​*ρc*​​*Vc*​=*Tc*​*To*​​*Vc*​⇒*Vo*​=*Vc*​*Tc*​*To*​​(3)**

**5. Energy Equation**

**For steady flow with heat addition (q*q*):**

**q=Cp(To−Tc)+Vo2−Vc22(4)*q*=*Cp*​(*To*​−*Tc*​)+2*Vo*2​−*Vc*2​​(4)**

**Substitute Vo=VcToTc*Vo*​=*Vc*​*Tc*​*To*​​ from Equation (3) into Equation (4):**

**q=Cp(To−Tc)+Vc22(To2Tc2−1)*q*=*Cp*​(*To*​−*Tc*​)+2*Vc*2​​(*Tc*2​*To*2​​−1)**

**Let To=θTc*To*​=*θTc*​ (where θ=To/Tc*θ*=*To*​/*Tc*​ is the temperature ratio):**

**q=CpTc(θ−1)+Vc22(θ2−1)(5)*q*=*Cp*​*Tc*​(*θ*−1)+2*Vc*2​​(*θ*2−1)(5)**

**6. Solve for θ*θ* (Temperature Ratio)**

**Rearrange Equation (5) to solve for θ*θ*:**

**Vc22θ2+CpTcθ−(CpTc+Vc22+q)=02*Vc*2​​*θ*2+*Cp*​*Tc*​*θ*−(*Cp*​*Tc*​+2*Vc*2​​+*q*)=0**

**This is a quadratic equation in θ*θ*:**

**θ=−CpTc±(CpTc)2+2Vc2(CpTc+Vc22+q)Vc2(6)*θ*=*Vc*2​−*Cp*​*Tc*​±(*Cp*​*Tc*​)2+2*Vc*2​(*Cp*​*Tc*​+2*Vc*2​​+*q*)​​(6)**

**Only the positive root is physically meaningful.**

**7. Final Relations**

1. **Exit Temperature:**

**To=θTc*To*​=*θTc*​**

1. **Exit Velocity:**

**Vo=Vcθ*Vo*​=*Vc*​*θ***

1. **Exit Density:**

**ρo=ρcTcTo=ρcθ*ρo*​=*ρc*​*To*​*Tc*​​=*θρc*​​**

**8. Summary of Key Equations**

| **Parameter** | **Equation** |
| --- | --- |
| **Temperature Ratio (θ*θ*)** | **Solve quadratic Equation (6).** |
| **Exit Velocity (Vo*Vo*​)** | **Vo=Vcθ*Vo*​=*Vc*​*θ*** |
| **Exit Temperature (To*To*​)** | **To=θTc*To*​=*θTc*​** |
| **Exit Density (ρo*ρo*​)** | **ρo=ρcθ*ρo*​=*θρc*​​** |

**9. Example Calculation**

**Assume:**

* **Tc=300 K, Vc=50 m/s, Cp=1005 *Tc*​=300K,*Vc*​=50m/s,*Cp*​=1005**

**Step 1: Solve for θ*θ* using Equation (6)**

**θ=−1005⋅300+(1005⋅300)2+2⋅502⋅(1005⋅300+5022+800×103)502*θ*=502−1005⋅300+(1005⋅300)2+2⋅502⋅(1005⋅300+2502​+800×103)​​θ≈2.5*θ*≈2.5**

**Step 2: Calculate To,Vo,ρo*To*​,*Vo*​,*ρo*​**

**To=300⋅2.5=750 K,Vo=50⋅2.5=125 m/s,ρo=ρc2.5*To*​=300⋅2.5=750K,*Vo*​=50⋅2.5=125m/s,*ρo*​=2.5*ρc*​​**

**10. Key Assumptions**

1. **Constant Pressure: Po=Pc*Po*​=*Pc*​.**
2. **Steady Flow: No time-dependent changes.**
3. **Ideal Gas Behavior: Valid for air and combustion products.**

**11. References**

* **Gas Turbine Theory by H.I.H. Saravanamuttoo, G.F.C. Rogers, and H. Cohen.**
* **Thermodynamics: An Engineering Approach by Yunus A. Çengel and Michael A. Boles.**

**Let me know if you need numerical calculations for specific parameters!**